Scan Time Effect on Load IAE

The equation for the integrated absolute error (IAE) as a function of controller gain (K_c) and reset time (T_i) derived from the PI controller's response to a load upset is.

$$IAE = \frac{T_i + \Delta t_s}{K_p * K_c} * E_o \qquad (1)$$

The Lambda tuning equation to set a degree of transfer of variability in terms of an original dead time (θ_o) is:

$$K_c = \frac{T_i}{K_p * (\lambda_f * \tau_1 + \theta_o)} \qquad (2)$$

The SIMCA and Ziegler Nichols (ZN) tuning equation for maximum transfer of variability in terms of a new dead time (θ_n) for $\theta_n < \tau_1$ is:

$$K_c = 0.5 * \frac{\tau_1}{K_p * \theta_n} \qquad (3)$$

A detuned controller (e.g. lower controller gain) has the same load rejection capability as a loop with more dead time. The maximum performance for a given dead time is suggested by using the SIMC/ZN tuning for maximum controller gain in equation 1 for the IAE. Thus, we can find out how much dead time is implied in a detuned controller by setting the detuned controller gain equal to the equation for the SIMC/ZN controller gain.

If you set Lambda equation equal to the SIMC/ZN equation, set the reset time equal to the time constant ($T_i = \tau_1$), and cancel terms, then you end up with the following equation for the new dead time (θ_n) as a function of the Lambda factor (λ_f), the time constant (τ_1), and the original dead time (θ_o).

$$\theta_n = 0.5 * (\lambda_f * \tau_1 + \theta_o) \qquad (4)$$

If we take the worst case where the load upset arrives just after the analog input block reads the measurement, all of the scan time (Δt_s) becomes dead time. In this case, the new dead time is equal to the original dead time plus the scan time.

$$\theta_n = \theta_o + \Delta t_s \qquad (5)$$

If you set equation 4 equal to equation 5, then the scan time (Δt_s) for $\theta_o < \lambda_f * \tau_1$ is:

$$\Delta t_s = 0.5 * (\lambda_f * \tau_1 - \theta_o) \qquad (6)$$