New Directions in Bioprocess Modeling and Control
Appendix C – Unification of Controller Tuning Relationships
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There is an intensive search for a unified field theory that would bring together quantum and gravitational forces and provide the underlying truth in physical laws. This appendix shows we are fortunate enough to have achieved a unification of Lambda, Internal Model Control, and Ziegler Nichols reaction curve and ultimate oscillation tuning methods for bioprocess control. For vessel temperature, concentration, and gas pressure control, the controller tuning equations from diverse methods reduce to a common form, where the maximum controller gain (Equation 3-3f in Chapter 3) is proportional to the time constant to dead time ratio (τ_1 / τ_d) and is inversely proportional to the open loop gain (K_o), commonly known as the process gain. This common form is easy to remember and provides insight as to the relative effects of process dynamics on tuning and hence on loop performance. Appendix C concludes with a derivation of the equation to predict the control error (integrated absolute error) in terms of the tuning settings (Equation 2-2a in Chapter 2) from the response of a PI controller to load disturbances.

Lambda tuning provides stable results for any Lambda value. Normally Lambda is set large enough to provide the degree of slowness desired to reduce interaction and promote the coordination of loops. If Lambda is set much smaller than normally expected as outlined below, the result is the common form, which provides maximum disturbance rejection and minimum integrated absolute error. Thus, Lambda tuning has the advantage of the user being able to set the degree of transfer of variability from the process output to process input to achieve a variety of objectives.

The Lambda tuning equations for self regulating processes are as follows:

$$\lambda = \lambda_f * \tau_1 \qquad \text{(C-1a)}$$

$$T_i = \tau_1 \qquad \text{(C-1b)}$$

$$K_c = \frac{T_i}{K_o * (\lambda + \tau_d)} \qquad \text{(C-1c)}$$

$$T_d = \tau_2 \qquad \text{(C-1d)}$$

$$K_o = K_{mv} * K_{pv} * K_{pv} \qquad \text{(C-1e)}$$

The Lambda factor is the ratio of closed loop time constant (Figure 3-3a) to the open loop time (Figure 3-3b) where the open loop time constant is the largest time constant (τ_1). For maximum load rejection capability, a Lambda equal to the total loop dead time ($\lambda = \tau_d$) can be used and the loop will still be stable if the dynamics are accurately known. For many temperature loops, this corresponds roughly to a 0.1 Lambda factor ($\lambda_f = 0.1$). If we substitute this small Lambda into Equation C-1c, we end up with the Simplified Internal

Model Control (SIMC) Equation C-1f (Equation 3-3f in Chapter 3), which has recently been documented to provide the best load rejection (tightest control) for self-regulating processes [1]. Until recently, proportional band (PB) was predominantly used instead of controller gain. The equation for minimum proportional band is 100% times the inverse of Equation C-1f since the controller gain is 100% divided by the proportional band (K_c =100% / PB). Proportional band is the % change in the control error that will cause a 100% change in controller output from the proportional mode.

$$K_c = 0.5 * \frac{\tau_1}{K_o * \tau_d} \tag{C-1f}$$

Equation C-1f, which is the Equation 3-3f in Chapter 3, has dominated the literature since the days of Ziegler Nichols. The multiplier ranges from 0.4 to 0.8 and the exponent of the time constant to dead time ratio varies from 0.9 to 1.0. The differences in the multipliers or exponents are insignificant because in practice the user has backed off from the maximum gain and the effect of errors or changes commonly seen in the identified process gain, time constant, and dead time is larger than the effect of the coefficients.

If we substitute the definition of a pseudo or "near" integrator gain per Equation C-1g into Equation C-1f we end up with the Equation C-1h, which is the SIMC tuning shown to provide the tightest control of integrating processes [1].

$$K_i = \frac{K_o}{\tau_1} \tag{C-1g}$$

$$K_c = 0.5 * \frac{1}{K_i * \tau_d} \tag{C-1h}$$

If we make a change in nomenclature where the integrating process gain is the reaction rate (K_i =R) and the dead time is the delay time (τ_d =L) we end up with the controller gain per the Ziegler Nichols "reaction curve" method developed in the 1940s.

The "reaction curve" method is suitable for self-regulating processes with large time constants and integrating processes. However, the documentation of the "reaction curve" method showed the process was lined out ($CV_1 / \Delta t = 0$) just before the change is made in the controller output so that R could be computed from just the ramp rate of the process variable after the change. This may not be the case, especially for integrating processes, since the controller is in manual. Therefore it is critical to take into account the change in ramp rates from "before" to "after" the change and use % rather than engineering units as shown in Equation C-1i. The "short cut" method in the book titled *Good Tuning – A Pocket Guide* provides a detailed procedure for the use of Equation C-1i and the correction of the observed dead time for the effect of final element resolution or dead band to provide a quick estimate of the controller tuning for slow processes [2].

$$K_i = \frac{CV_2/\Delta t - CV_1/\Delta t}{\Delta CO} \qquad \text{(C-1i)}$$

It may be difficult to accurately identify the second largest time constant. Therefore, the Internal Model Control computation of the derivative time as shown in Equation C-1j may be useful. For a first order (single time constant) approximation of a concentration or temperature response, about half of the total loop dead time originates from the second largest time constant as shown in Equation C-1k. Since in this case the time constant is much larger than the dead time, the dead time term in the denominator of Equation C-1j becomes just twice the largest time constant ($2*\tau_1$). If you then cancel out the time constant in the numerator and denominator, you end up with the derivative time approximately equal to half the dead time. Equation C-1j coupled with Equation C-1k reduces to Equation C-1d.

$$T_d = \frac{\tau_1 * \tau_d}{2 * \tau_1 + \tau_d} \qquad \text{(C-1j)}$$

For interactive lags found in temperature, concentration, and gas pressure processes:

$$\tau_2 = 0.5 * \tau_d \qquad \text{(C-1k)}$$

The Lambda tuning equations for integrating processes are as follows:

$$\lambda = \lambda_f \ / \ K_i \qquad \text{(C-1l)}$$

$$T_i = 2 * \lambda + \tau_d \qquad \text{(C-1m)}$$

$$K_c = \frac{T_i}{K_i * (\lambda + \tau_d)^2} \qquad \text{(C-1n)}$$

The Lambda factor is the ratio of closed loop arrest time to open loop arrest time where the open loop arrest time is simply the inverse of the integrating process gain ($1/K_i$). For maximum load rejection capability, a Lambda equal to the total loop dead time ($\lambda = \tau_d$) can again be used and the loop will still be stable if the dynamics are accurately known. In this case, Equation C-1n reduces to Equation C-1h but with a multiplier of 0.75 instead of 0.5. The higher multiplier is insignificant since it is rarely desirable and dynamics are seldom known accurately enough to take advantage of this increase in the controller gain.

For many processes with a true or near an integrating response, the controller gains computed by Equation C-1h are much higher than desired or needed particularly for bioreactors since the disturbances are so slow. However, a much lower controller gain can lead to nearly sustained oscillations with a very long period. To prevent this from occurring, Equation C-1o which is developed from the transfer function for the closed loop response of an integrating process can be used to insure the response is overdamped.

The integral time to insure an overdamped response in integrating processes:

$$T_i > \frac{4}{K_i * K_c} \qquad \text{(C-1o)}$$

For self-regulating processes, the transfer function for the closed loop response yields Equation 3-3p for an overdamped response. The product of the controller gain and the process gain is much larger than one for temperature and concentration loops on well mixed volumes because the time constant is so large, which means you can cancel out the product of gains in the numerator. If you then use Equation C-1g to get an equivalent process integrating gain, Equation C-1p reduces to Equation C-1o.

The integral time to insure an overdamped response in self-regulating processes:

$$T_i > \frac{4 * (K_o * K_c * \tau_1)}{(1 + K_o * K_c)^2} \qquad \text{(C-1p)}$$

Ziegler Nichols developed controller tuning equations based on field measurements of the ultimate gain and ultimate period. For a manual tuning test, the derivative time is set to zero and the integral time is set at least 10 times larger than normal so that most of the controller response is from the proportional mode. The controller gain is then increased to create equal sustained oscillations. The controller gain at this point is the ultimate gain and the oscillation period is the ultimate period. In industry, the gain is only increased until decaying oscillations first appear to reduce the disruption to the process. Auto tuners and adaptive controllers make this manual controller tuning unnecessary. The “relay method” is extensively employed by “on-demand” auto tuners to automatically compute the ultimate period and gain by switching the controller output when it crosses and departs from a noise band centered on the set point [2] [3].

The ultimate gain for self-regulating processes per the amplitude ratio is [3]:

$$K_u = \frac{[1 + (\tau_1 * 2 * \pi / T_u)^2]^{0.5}}{K_o} \qquad \text{(C-1q)}$$

For the Ziegler Nichols ultimate oscillation method, the controller gain is simply a fraction of the ultimate gain and the integral (reset) time is a fraction of the ultimate period as follows for a PI controller:

$$K_c = 0.4 * K_u \qquad \text{(C-1r)}$$

$$T_i = 0.8 * T_u \qquad \text{(C-1s)}$$

If you also take into account that the squared expression in the numerator of Equation C-1q is much larger than one, you end up with Equation C-1t. For most temperature and composition loops on bioreactors, the ultimate period is approximately 4 times the dead time ($T_u = 4*\tau_d$). If you substitute this relationship into C-1s, and use Equation C-1q, you end up with Equation C-1u. After multiplication of numerical factors Equation C-1u become Equation C-1v (Equation C-1f with a slightly larger multiplier).

$$K_c = 0.4* \frac{(\tau_1 * 2 * \pi) / T_u}{K_o} \qquad \text{(C-1t)}$$

$$K_c = 0.4* \frac{(\tau_1 * 2 * \pi) / (4 * \tau_d)}{K_o} \qquad \text{(C-1u)}$$

$$K_c = 0.6 * \frac{\tau_1}{K_o * \tau_d} \qquad \text{(C-1v)}$$

If the ultimate period is about 4 times the dead time ($T_u = 4*\tau_d$), then the integral time ends up as about 3 times the dead time per Equation 3-1s for the ultimate oscillation method, which is the same result you get per Equation C-1m for the Lambda tuning method when Lambda is reduced to equal the dead time. This reset time is generally considered to be too fast. The SIMCA method states that while 4 times the dead time provides the best performance and an increase to 8 times the dead time provides better robustness. If 4 times the dead time is used, then Equations C-1d and C-1k result in a reset time that is 8 times the rate time setting. While most of the literature shows the rate time equal to ¼ the reset time, in practice a rate time that is 1/8 to 1/10 of the reset time provides a smoother response.

The Equation 2-2a in Chapter 2 for the integrated absolute error (IAE) can be derived from the response of a PI controller to a load upset [5]. The module execution time (Δt) is added to the reset or integral time (T_i) to show the effect of how the integral mode is implemented in some digital controllers. An integral time of zero ends up as a minimum integral time equal to the execution time so there is not a zero in the denominator of Equation C-1w. For analog controllers, the execution time is effectively zero [6].

$$\Delta CO = K_c * \Delta E_t + [K_c / (T_i + \Delta t)] * \text{Integral } (E_t * \Delta t) \quad \text{(C-1w)}$$

The errors before the disturbance and after the controller has completely compensated for the disturbance are zero ($\Delta E_t = 0$). Therefore, the long term effect of the proportional mode, which is first term in Equation C-1w, is zero. Equation C-1w reduces to Equation C-1x [5].

$$\Delta CO = [(K_c / (T_i + \Delta t)] * \text{Integral } (E_t * \Delta t) \quad \text{(C-1x)}$$

For an over damped response:

$$IAE = \text{Integral } (E_t * \Delta t\ ) \quad \text{(C-1y)}$$

The open loop error is the peak error for a step disturbance for the case where the controller is in manual (loop is open). The open loop error (E_o) is the open loop gain (K_o) times the shift in controller output (ΔCO) required to compensate for the disturbance when the controller is auto (loop is closed).

$$E_o = K_o * \Delta CO \quad \text{(C-1z)}$$

Equation C-1x solved for the IAE defined in Equation C-1y and the open loop error defined in Equation C-1z becomes Equation C-1aa. If you ignore the effect of module execution time (Δt) on the integral mode, Equation 2-2a in Chapter 2 is Equation C-1aa for an over damped response because the integrated absolute error (IAE) is the same as the integrated error (E_i). Even for a slightly oscillatory response, the approximation has proven to be close enough [4].

$$IAE = \frac{1}{(K_o * K_c\ )} * (T_i + \Delta t) * E_o \quad \text{(C-1aa)}$$

For vessel temperature, concentration, and pressure control, we can use Equation C-1f for the maximum controller gain and 4 times the dead time for the minimum reset time to express the minimum integrated absolute error in terms of the process dynamics. The resulting Equation C-1bb shows that the minimum integrated absolute error is proportional to the dead time squared for tuning settings that give the tightest control. Note that the open loop gain has cancelled out. This equation can be independently derived by multiplying the peak error for a step disturbance by the dead time [3][6].

$$IAE = 2 * \frac{\tau_d}{\tau_1} * (4*\tau_d\ + \Delta t) * E_o \quad \text{(C-1bb)}$$

In practice, controllers are not tuned this aggressively. Often the reset time is set equal to the time constant and a Lambda factor of 1.0 is used, which corresponds to a controller gain that is about ten times smaller than the maximum controller gain for a bioreactor's primary loops, such as composition, pressure, and temperature.

If the disturbance is not a step change, the integrated absolute error will be smaller. The effect of a slow disturbance can be approximated by adding the disturbance time constant to the open loop time constant (τ_1) in the denominator of Equation C-1bb.

An increase in the module execution time shows up as an increase in the loop dead time for unmeasured disturbances. If the disturbance arrives immediately after the process variable is read as an input to the module, the additional dead time is about equal to the module execution time. If the disturbance arrives immediately before the process variable is read, the additional dead time is nearly zero. On the average, the additional dead time can be approximated as 50% of the module execution time. Simulations that create a disturbance that is coincident with the controller execution will not show much of an effect of execution time on performance. This scenario misleads users into thinking the execution time of model predictive control is not important for load rejection. For chromatographs where the result is only available for transmission after the processing and analysis cycle, the additional dead time is 150% of the analyzer cycle time [2][3][5].

Equation C-1bb shows the effect of the largest time constant, loop dead time, and module execution time on absolute integrated error if the controller is always retuned for maximum performance. A detuned controller may not do much better than a tightly tuned controller for a larger loop dead time or module execution time [5]. Thus, the value of reducing these delay times depends upon the controller gain used in practice. For example, the controller gain is simply the inverse of the open loop gain for a Lambda factor of one in a loop with a dead time much smaller than the time constant. In other words, Equation C-1c reduces to Equation C-1cc.

$$K_c = \frac{1}{K_o} \tag{C-1cc}$$

If you substitute Equation C-1cc into Equation C-1f and solve for the dead time, you end up with Equation C-1dd. This shows a Lambda factor of one on a primary reactor loop implies a dead time that is about ½ of the time constant. The integrated absolute error for this case will not appreciably increase until the dead time is about ½ of the time constant. Thus, time and money spend on reducing the dead time or module execution time below this implied dead time has little value unless the controller is retuned [5].

$$\tau_d = 0.5 * \tau_1 \tag{C-1dd}$$

Nomenclature

ΔCO = shift in controller output to compensate for disturbance (%)
E_i = integrated error (% seconds)
E_o = open loop error for a step disturbance (%)
E_t = error between SP and PV during the disturbance
IAE = integrated absolute error from the disturbance (% seconds)
K_c = controller gain (dimensionless)
K_i = integrating gain (%/sec/% or 1/sec)
K_o = open loop gain (dimensionless)
K_u = ultimate gain (dimensionless)
λ = Lambda (closed loop time constant or arrest time) (sec)
λ_f = Lambda factor (ratio of closed to open loop time constant or arrest time) (dimensionless)
Δt = module execution time (sec)
τ_d = total loop dead time (sec)
τ_1 = largest open loop time constant (sec)
τ_2 = second largest open loop time constant (sec)
T_i = integral (reset) time setting (sec/repeat)
T_d = derivative (rate) time setting (sec)
T_u = ultimate period (sec)

References

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